ECE209AS (Fall 2025)

Computational Robotics

Prof. Ankur Mehta ⟨mehtank@ucla.edu⟩

|

Show all work.

1 Linear algebra

1.1 Consider the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 3 & 6 & 9 & 5 \\ 2 & 4 & 6 & 9 \end{bmatrix}$$

1(a). **What is the rank of A?**

1(b). **For what b_3 does the equation $Ax = b = \begin{bmatrix} 1 \\ 2 \\ b_3 \end{bmatrix}$ have a solution?**

1(c). **What is the complete (general) solution x such that $Ax = b$, with b as above?**

1.2 How difficult did you find these problems (easy / needed review / hard)?

§

2 Differential equations

2.1 Consider the following coupled oscillator with input $u(t)$ and output $y(t)$:

$$\dot{y}(t) = z(t) + u(t) \quad (1)$$
$$\dot{z}(t) = 2y(t) + z(t) + 3u(t) \quad (2)$$

2(a). **Rewrite the system with a state-space description:**

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + Bu$$
$$y = C\boldsymbol{x} + Du$$

What are the values of $A, B, C, D, \boldsymbol{x}$?

2(b). **Describe the normal modes of the system and their stability.**

2(c). **Consider a feedback controller of the form $u(t) = ky(t)$. For what values of k is the resulting system stable?**

2.2 How difficult did you find these problems (easy / needed review / hard)?

§

3 Probability

3.1 Flip a coin 8 times.

Let's say you obtained the sequence: **T T T H T T H T**

3(a). **If the coin is perfectly fair, what is the probability of obtaining this sequence?**

3(b). **If the coin is biased with a probability of heads p(H) = $\frac{1}{4}$, what is the probability of obtaining this sequence?**

3(c). **Let's say you have 4 coins, 3 of which are fair and 1 is biased as above. You pick one coin uniformly at random from those. What is the likelihood (probability) that the coin you've picked is biased?**

3(d). **You flip this unknown coin 8 times to obtain the above sequence. Now what is the likelihood (probability) that the coin you've picked is biased?**

3.2 How difficult did you find these problems (easy / needed review / hard)?

§

4 Programming

4.1 Consider the coins from problem 3.

We will simulate flipping a coin 100 times to try to guess which coin we've picked, refining our estimate of the likelihood after each flip's outcome.

4(a). **Write a function that takes in 2 parameters:**

- **the type of coin (fair or biased), and**
- **the number of flips;**

and returns the resulting simulated sequence of outcomes. Generate 10 sequences of 40 flips each: 5 sequences for a fair coin and 5 sequences for a biased coin.

4(b). **Modify that function to also calculate the likelihood that the coin is biased after each successive flip, given the conditions of problem 3(c).**

4(c). **Generate a properly labeled graph plotting the likelihood of having picked a biased coin as it evolves after each of 100 simulated flips of a fair coin. Overlay 5 independent simulations on the same graph.**

4(d). **Generate a similar graph, this time plotting 5 independent runs of simulating 100 flips of a biased coin.**

4.2 How difficult did you find these problems (easy / needed review / hard)?

§

5 Summary

5.1 How long did this pset take you?

Please note that your answers to this question and the questions about difficulty are solely for me to assess and adapt my problem sets and lectures to better serve you. Please answer truthfully; your responses do not in any way affect your grade.